Politehnica University of Timișoara
Department of Mathematics

CONFERENCE
ON ULAM'S TYPE STABILITY

October 8–13, 2018
Timișoara, Romania

Dear Colleagues,

Welcome to the Politehnica University of Timișoara, and to the

Conference on Ulam's Type Stability.

The meeting has been organized by the Department of Mathematics of Politehnica University of Timișoara in cooperation with Department of Mathematics of the Pedagogical University of Cracow (Poland) and Faculty of Applied Mathematics of AGH University of Science and Technology (Cracow, Poland). We focus on various investigations motivated by the notion of Hyers-Ulam stability.

We do hope that you will spend a fruitful and pleasant time here.

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General Information

The Conference on Ulam's Type Stability (CUTS) is being held at the *Conference Centre - Library* of the Politehnica University of Timişoara, Romania (Vasile Parvan Boulevard, no. 2B). Every guest will be hosted at the university hotel *Casa Politehnicii 2* (Mihai Eminescu Boulevard, no. 2, Timişoara). It is situated at 10 minutes walking distance to the University's Conference Center and 5 minutes to the city center.

The meals are served at the following times

$\mathbf{7^{30} – 9^{00}}$	Breakfast
$\mathbf{13^{00}}$	Lunch
$\mathbf{19^{00}}$	Dinner (banquet on Thursday, wellcome dinner on Wednesday, at the *Sports Center no. 2* of the Politehnica University of Timişoara, adress: Păunescu Podeanu Street, no. 2.)

A guided tour of Timişoara is planned on Wednesday afternoon. Among others, we will be wandering through some famous sights of our city. Please let us know if you intend to take part in the tour by signing up on a list at the reception of the hotel until 15^{00} on Tuesday, 9th of October. There is no additional fee. The departure is due to take place at 15^{00} from the hotel. The last stop of our tour will be Sports Center no. 2 (belonging to the Politehnica University of Timişoara), where a welcome dinner is awaiting for you.

Please do not hesitate to contact the Organizing Committee if you need any assistance.

The opening of the CUTS 2018 will be held on Tuesday, October 9, at 9^{30} in the "Polyvalent Hall" of the Politehnica University of Timişoara Conference Centre - Library (Vasile Pârvan Boulevard, no. 2B). All the scientific sessions will take place in this room. During the conference 13 sessions are planned, with a total of 20 talks. Morning sessions start at 9^{30} and afternoon sessions start at 16^{00}. There are no afternoon sessions on Wednesday, October 10.

For each regular talk the speaker is given 20 minutes and extra 10 minutes are scheduled for discussion. There are also 7 plenary lectures 50 minutes long with 10 minutes for discussion. In each day (exception Wednesday) *Problems and Remarks* sessions are scheduled.

Within the sessions there will be no breaks between talks, therefore the schedule of the individual talks is approximate. Please contact us if you have special wishes concerning your talk or you need any assistance. The lecture room is equipped with a computer and a data projector. Please, make a copy of the file of your talk on the computer in advance.

The report from the conference will be published on the web site of the conference *http://cuts.up.krakow.pl* . It will include abstracts of delivered talks, problems and remarks and the list of participants with addresses. The report will be prepared on the basis of the present booklet. Therefore, if you wish to make any corrections please insert them in the copy of this booklet which will be placed in the lecture room. Those who contribute to *Problems and Remarks*' sessions are kindly requested to prepare a note on their presentation and give it to one of the scientific secretaries as soon as possible but before the end of the conference. For future correspondence concerning the report please use one of the addresses: *cuts@up.krakow.pl* and *zlesniak@up.krakow.pl* .

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Invited lectures

1. **Janusz Brzdęk**, *On some fixed point results for the complete b–metric spaces*
2. **Erdal Karapınar**, *Retrospective approach to the metric fixed point theory*
3. **Zsolt Páles**, *Monomial selections of set-valued functions*
4. **Adrian Petruşel**, *Ulam-Hyers stability in fixed point theory*
5. **Dorian Popa**, *Best constant in Ulam stability*
6. **Ioan Raşa**, *Ulam stability for composition of operators*
7. **László Székelyhidi**, *Stability of functional equations related to spherical functions*

Regular lectures

1. **Alina-Ramona Baiaş**, *On Ulam stability of a linear difference equation*
2. **Constantin Buşe**, *Hyers-Ulam stability for equations with differences and differential equations with time dependent and periodic coefficients*
3. **Jacek Chmieliński**, *Stability of the orthogonality reversing property*
4. **Liliana Guran**, *Ulam-Hyers stability problems for fixed point theorems on generalized metric spaces*
5. **Daniela Inoan**, *On Ulam stability of linear differential systems*
6. **Zbigniew Leśniak**, *Fixed points of a linear operator of polynomial form*
7. **Renata Malejki**, *On the stability of a generalized Fréchet functional equation*
8. **Daniela Marian**, *Ulam-Hyers-Rassias stability of some quasilinear partial differential equations of first order*
9. **Diana Otrocol**, *Ulam stability of a linear difference equation in topological vector spaces*
10. **Paweł Pasteczka**, *On the quasi-arithmetic Gauss-type iteration*
11. **Adrian Viorel**, *On the conditional Ulam stability of nonlinear evolution equations*
12. **Peter Volkmann**, *Solutions of ordinary differential equations in closed subsets of a Banach space*
13. **Pavol Zlatoš**, *A Uniform Stability Principle for Dual Lattices*

Alina-Ramona Baiaş
Technical University of Cluj-Napoca, Cluj-Napoca, Romania
(joint work with **Dorian Popa**)

On Ulam stability of a linear difference equation

In this paper we obtain a result on Ulam stability and on the best Ulam constant for the linear difference equation $x_{n+2} = ax_{n+1} + bx_n$, where $a, b \in \mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$ and $(x_n)_{n\geq 0}$ is a sequence in a Banach space X over the field $\mathbb{K}$. In this way we improve and complement some recent results on Ulam stability of the first and second order linear difference equations with constant coefficients.

Janusz Brzdęk
AGH University of Science and Technology, Kraków, Poland

On some fixed point results for the complete b–metric spaces

Let us recall that (Y, d, η) is a b–metric space provided Y is a nonempty set, $\eta \geq 1$ is a fixed real number and $d\colon Y \times Y \to [0, +\infty)$ is a function satisfying the following three conditions:

(A) $d(x, y) = 0$ if and only if $x = y$;

(B) $d(x, y) = d(y, x)$;

(C) $d(x, y) \leq \eta\big(d(x, z) + d(z, y)\big)$

for all $x, y, z \in Y$.

Motivated by several outcomes in [1, 2, 1, 4, 5, 6], we discuss some possible fixed point results for classes of functions taking values in a b-metric space, motivated by the Ulam stability issues.

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Constantin Buşe

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(joint work with **Vasile Lupulescu** and **Donald O'Regan**)

Hyers-Ulam stability for equations with differences and differential equations with time dependent and periodic coefficients

Let q be a positive integer and let (a_n) and (b_n) be two given $\mathbb{C}$- valued and q-periodic sequences. First we prove that the linear recurrence in $\mathbb{C}$

$$x_{n+2} = a_n x_{n+1} + b_n x_n, \quad n \in \mathbb{Z}_+$$

is Hyers-Ulam stable if and only if the spectrum of the monodromy matrix $T_q := A_{q-1} \cdots A_0$ ((i.e. the set of all its eigenvalues) does not intersect the unit circle $\Gamma = \{z \in \mathbb{C} : |z| = 1\}$, i.e. T_q is hyperbolic. Here (and in as follows) we let

$$A_n = \begin{pmatrix} 0 & 1 \\ b_n & a_n \end{pmatrix} \quad n \in \mathbb{Z}_+.$$

Second we prove that the linear differential equation

$$x''(t) = a(t)x'(t) + b(t)x(t), \quad t \in \mathbb{R},$$

(where $a(t)$ and $b(t)$ are $\mathbb{C}$-valued continuous and 1-periodic functions defined on $\mathbb{R}$) is Hyers-Ulam stable if and only if $P(1)$ is hyperbolic; here $P(t)$ denotes the solution of the first order matrix 2-dimensional differential system

$$X'(t) = A(t)X(t), \quad t \in \mathbb{R}, \quad X(0) = I_2,$$

where I_2 is the identity matrix of order 2 and

$$A(t)A(t) = \begin{pmatrix} 0 & 1 \\ b(t) & a(t) \end{pmatrix}, \quad t \in \mathbb{R}.$$

Jacek Chmieliński

Pedagogical University of Cracow, Kraków, Poland
(joint work with **Paweł Wójcik**)

Stability of the orthogonality reversing property

For a linear operator $T\colon X \to X$, acting on a real normed space X, we consider the *orthogonality reversing property* (introduced in [2]):

$$x \perp_{\mathrm{B}} y \quad \Longrightarrow \quad Ty \perp_{\mathrm{B}} Tx, \qquad (x, y \in X),$$

where $\perp_{\mathrm{B}}$ stands for the (nonsymmetric) Birkhoff orthogonality.

Applying a definition of an *approximate Birkhoff orthogonality* $\perp_{\mathrm{B}}^{\varepsilon}$ (introduced in [1] with a new characterization recently given in [3]) we define the class of linear operators *approximately reversing orthogonality* as those satisfying the property:

$$x \perp_{\mathrm{B}} y \quad \Longrightarrow \quad Ty \perp_{\mathrm{B}}^{\varepsilon} Tx, \qquad (x, y \in X).$$

Then, we study the stability of the considered property (cf. [5]).

A related problem of an *approximate symmetry* of the Birkhoff orthogonality will be also discussed (cf. [4]).

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Liliana Guran

Vasile Goldiş Western University of Arad, Arad, Romania
(joint work with **Monica-Felicia Bota** and **Khurram Shabbib**)

Ulam-Hyers Stability Problems for Fixed Point Theorems on Generalized Metric Spaces

The purpose of this paper is to present some fixed point results in generalized metric spaces in Perov's sense using a contractive condition of Hardy-Rogers type. The data dependence of the fixed point set, the well-posedness of the fixed point problem, as well as, the Ulam-Hyres stability are also studied.

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Daniela Inoan

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(joint work with **Dorian Popa** and **Ioan Raşa**)

On Ulam stability of linear differential systems

In this talk we present some results on generalized Ulam stability for a linear system of differential equations with nonconstant coefficients. To this purpose we obtain first some bounds for a solution of a linear system using a Bernoulli type inequality. The main result is then applied to obtain stability results for some second order linear differential equations.

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Erdal Karapınar

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Retrospective approach to the metric fixed point theory

The main goal of this talk is to indicate some indispensable notes on the recent results in the frame of the metric fixed point theory with a retrospective approach. Starting with a brief history of metric fixed point theory, we shall emphasize the mainstreams of the metric fixed point theory researches. After then, we shall show the equivalence of the metric fixed point results that were proved in the setting of different abstract spaces. The talk will be largely expository.

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Zbigniew Leśniak

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(joint work with **Janusz Brzdęk** and **El-sayed El-hady**)

Fixed points of a linear operator of polynomial form

We present a fixed-point theorem motivated by some issues arising in Ulam type stability.

For a linear operator $\mathcal{L} : X \to X$ satisfying the Lipschitz condition we define the operator $\mathcal{P} : X \to X$ by

$$\mathcal{P}\psi := p_3\mathcal{L}^3\psi + p_2\mathcal{L}^2\psi + p_1\mathcal{L}\psi, \qquad \psi \in X,$$

where X is an extended complex Banach space and $p_1, p_2, p_3 \in \mathbb{C}$, $p_3 \neq 0$.

Under some assumptions on the roots of the characteristic polynomial of the equation

$$p_3\mathcal{L}^3\psi + p_2\mathcal{L}^2\psi + p_1\mathcal{L}\psi = \psi, \tag{1}$$

we prove that for every $\varphi \in X$ such that $\big\|\mathcal{P}\varphi - \varphi\big\| < \infty$, the operator $\mathcal{P}$ has a unique fixed point $\psi \in X$ for which $\|\varphi - \psi\| < \infty$.

Using this fixed-point theorem we obtain a stability result for equation (1).

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Renata Malejki

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(joint work with **Anna Bahyrycz, Janusz Brzdęk, Eliza Jabłońska and Zbigniew Leśniak**)

On the stability of a generalized Fréchet functional equation

We present a generalization of the Fréchet functional equation, stemming from a characterization of the inner product spaces. We show, in particular, that under some weak additional assumptions each solution of such equation is additive. We also obtain a theorem on the Ulam type stability of the equation.

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Daniela Marian

Technical University of Cluj-Napoca, Cluj-Napoca, Romania
(joint work with **Nicolae Lungu**)

Ulam-Hyers-Rassias stability of some quasilinear partial differential equations of first order

In this paper we investigate the Ulam-Hyers-Rassias stability for some quasilinear partial differential equations.

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(joint work with **Adela Novac and Dorian Popa**)

Ulam stability of a linear difference equation in topological vector spaces

Let X be a sequentially complete Hausdorff locally convex space over $\mathbb{C}$. We obtain a result on Ulam stability for the linear difference equation

$$x_{n+p} = a_1 x_{n+p-1} + \ldots + a_p x_n,$$

where $a_1, \ldots, a_p \in \mathbb{C}$ and $x_0, \ldots, x_{p-1} \in X$.

Zsolt Páles

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Monomial selections of set-valued functions

Applying the vector-valued extension of the so-called invariant mean technique invented by L. Székelyhidi ([3]), we investigate the Hyers–Ulam stability of the generalized monomial functional inclusion

$$p_0 f(x) + p_1 f(xy) + \cdots + p_n f(xy^n) \in \Phi(y) \qquad (x, y \in X),$$

where $(X, \cdot)$ is a commutative semigroup, Y is a locally convex space over $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, $p_0, p_1, \ldots, p_n \in \mathbb{K}$ with $p_0 + p_1 + \cdots + p_n = 0$, $f : X \to Y$, and $\Phi : X \to 2^Y$ is a set-valued function with nonmpty weakly compact and convex values.

Our theorems generalize the results obtained in the papers [1] and [2].

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On the quasi-arithmetic Gauss-type iteration

For a sequence of continuous, monotone functions $f_1, \ldots, f_n : I \to \mathbb{R}$ (I is an interval) we define the mapping $M : I^n \to I^n$ as a Cartesian product of quasi-arithmetic means generated by f_j-s. It is known that, for every initial vector, the iteration sequence of this mapping tends to the diagonal of I^n.

We will prove that whenever all f_j-s are $\mathcal{C}^2$ with nowhere vanishing first derivative, then this convergence is quadratic. Furthermore, the limit $\frac{\mathrm{Var} M^{k+1}(v)}{(\mathrm{Var}\, M^k(v))^2}$ will be calculated in a nondegenerated case.

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Ulam-Hyers stability in fixed point theory

In this talk, we will present several results concerning Ulam-Hyers stability of the fixed point equation/inclusion in various contexts.

Dorian Popa

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Best constant in Ulam stability

Let A, B be normed spaces and $T : A \to B$ an operator. We say that T is Ulam stable if there exists a constant $K \geq 0$ such that for every $g \in T(A)$, $\varepsilon > 0$ and $f \in A$ with $\|Tf - g\| \leq \varepsilon$, there exists $f_0 \in A$, such that $Tf_0 = g$ and $\|f - f_0\| \leq K\varepsilon$.

We denote by K_T the infimum of all Ulam constants K of T. If K_T is an Ulam constant of T, then it is called the best Ulam constant of T.

In this paper we give some results on the best Ulam constant for some linear operators and some linear differential operators.

Ioan Raşa

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Ulam stability for composition of operators

This talk is devoted to Ulam stability for composition of linear operators on Banach spaces. After some general results we provide examples of strongly continuous semigroups of Ulam stable, respectively Ulam unstable, operators. In this context we state the following

PROBLEM. Let $B_n,\ n \geq 1$, be the classical Bernstein operators on the Banach space $C[0,1]$ endowed with the supremum norm. Consider the semigroup $(T(t))_{t\geq 0}$, defined by (see [1])

$$T(t)f = \lim_{n\to\infty} B_n^{[nt]}f,\ \ f \in C[0,1].$$

Let $t > 0$. Is $T(t)$ injective/surjective/Ulam stable?

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László Székelyhidi

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Stability of functional equations related to spherical functions

Spherical functions are the basic building bricks of spherical spectral synthesis introduced by the author recently. These functions play the role of exponential functions in the spherical setting. In this talk we present stabilitytype theorems for functional equations related to spherical functions. Our proofs are based on superstability-type methods and on the method of invariant means.

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(joint work with **Dorian Popa** and **Ioan Raşa**)

On the conditional Ulam stability of nonlinear evolution equations

The present contribution deals with nonlinear evolution equations and their stability with respect to bounded perturbations. More precisely, we are interested in the particular situation when, only for perturbations below a critical threshold, the difference between the solutions of the exact and perturbed equations scales linearly with the amplitude of the perturbation. Formally, this phenomenon is described by a new concept that we call conditional Ulam stability, in contrast to unconditional stability as previously studied in, e.g., [1].

As a first example, we consider the logistic equation

$$y' = y - y^2$$

and its perturbed version

$$y' = y - y^2 + f(t), \quad \text{with} \quad |f(t)| \leq \epsilon,$$

which proves to be conditionally Ulam stable (see [2]). Then, the analysis is extended to an infinite-dimensional model: the Fisher–KPP equation [3]

$$u_t = u_{xx} + u - u^2.$$

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Peter Volkmann

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(joint work with **Gerd Herzog**)

Solutions of ordinary differential equations in closed subsets of a Banach space

Let E be a real Banach space and denote by $\alpha(B)$ the Kuratowski measure of non-compactness of bounded sets $B \subseteq E$. Suppose $T > 0$ and $f = g + k$, where $g, k : [0,T] \times E \to E$ are bounded continuous functions, g satisfying a one-sided Lipschitz condition and k an α-Lipschitz condition. Moreover let M be a closed subset of E such that

$$\liminf_{h \searrow 0} \frac{1}{h} \operatorname{dist}(x + hf(t,x), M) = 0 \quad ((t,x) \in [0,T[\times M).$$

Then for every $(\tau, a) \in [0,T[\times M$ the initial value problem

$$u(\tau) = a, \quad u'(t) = f(t, u(t)) \quad (\tau \leq t \leq T)$$

has a solution $u : [\tau, T] \to M$.

Pavol Zlatoš

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A Uniform Stability Principle for Dual Lattices

We will present a highly uniform stability or "almost-near" theorem for dual lattices of vector lattices $L \subseteq \mathbb{R}^n$. More precisely, we show that, for a vector x from the linear span of a lattice $L \subseteq \mathbb{R}^n$ with the Minkowski's first successive minimum $\lambda_1(L) \geq \lambda > 0$ to be ε-close to some vector from the dual lattice $L^\star$ of L, it is enough that the euclidean inner products $u\,x$ are δ-close (with $\delta < 1/3$) to some integers for all vectors $u \in L$ satisfying $\|u\| \leq r$, where $r > 0$ depends on n, λ, δ and ε, only. The result is derived as a consequence of its nonstandard version, formulated in terms of finite elements and the equivalence relation of infinitesimal nearness on the nonstandard extension ${}^*\mathbb{R}^n$ of $\mathbb{R}^n$: If x is a finite vector from the internal linear span of an internal lattice $L \subseteq {}^*\mathbb{R}^n$, such that the inner product $u\,x$ is infinitesimally close to some integer for each finite vector $u \in L$, then x is already infinitesimally close to some vector y from the dual lattice $L^\star$. The results generalize earlier analogous results proved for integral vector lattices by M. Mačaj and the author in [1].

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